UNIT I: SCALES AND INTERVALS

The Octave; The Overtone Series; The Circle of Fifths; the 12 Chromatic Pitches

Long before composers were writing music that one would call “tonal”, Western musicians had accepted in theory and in practice octave identity, the notion that the interval formed by two pitches whose frequency ratio was 2:l was the fundamental interval in music and music theory, and that notes in this 2:l ratio were “of the same pitch” (or “pitch class”, as one sometimes says), merely in “different octaves”. They had also accepted, in theory at least, a 12-fold division of this octave, a total musical repertory of 12 distinct pitches and their octave multiples. The chromatic tones B♭, E♭, and F♯ occur in the earliest Gregorian chant and chant treatises, ca 900 AD, along with, of course, the 7 “diatonic” pitches – the white keys on the piano. By 1400 theorists were discussing the complete 12-note chromatic scale, and keyboards were built with a full chromatic pitch capability, at least in the middle octaves. The chromatic pitches were, of course, used for diatonic purposes: to transpose the modes, or to add raised leading tones to them.

The exact frequency ratios of these 12 pitches, a matter of obvious practical importance to someone building an organ or tuning a harpsichord, was the subject of great controversy for several hundred years. “Tuning and temperament” is a complicated topic, and we will discuss it here only in the most cursory fashion.

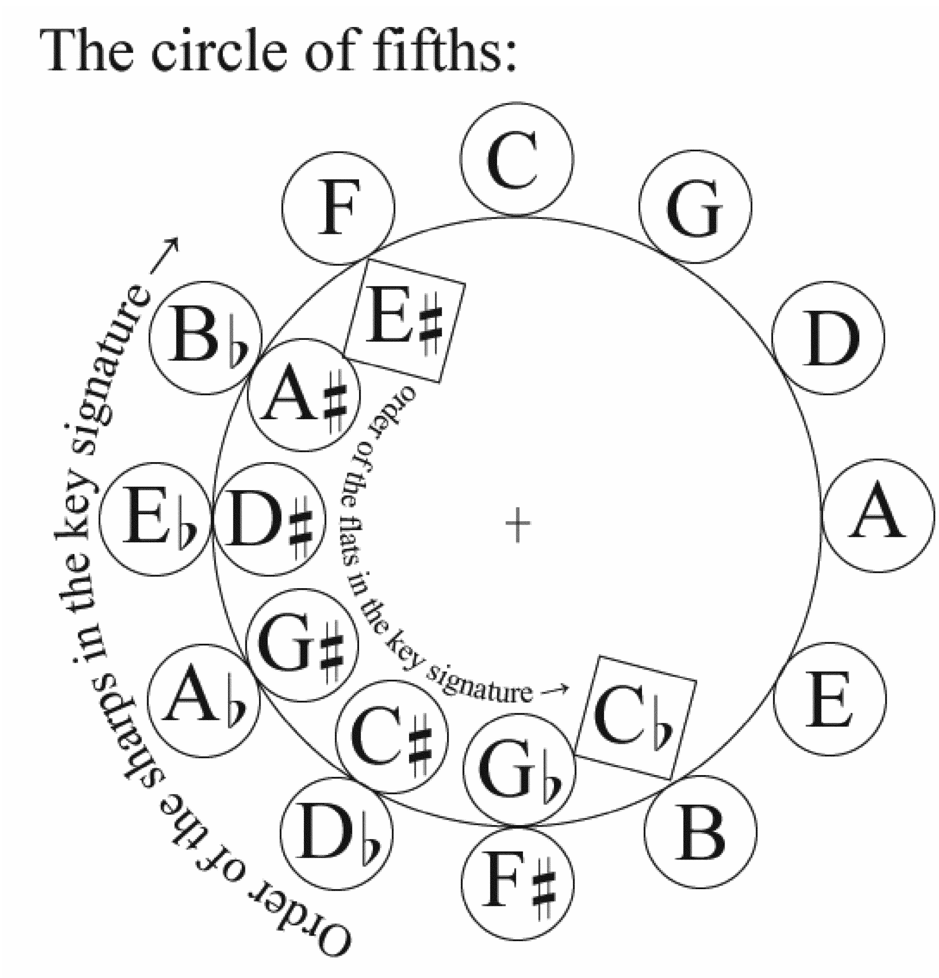
The first obvious suggestion is to determine the frequencies by using the *overtone series* – that is, the series of frequencies formed by successive integral multiplication of a given frequency, a phenomenon familiar to most brass and string players. If we take a given pitch – we won’t need to consider its actual vibration rate, so we can represent its frequency as simply 1 – and multiply that frequency by successive integers, we get the pitches shown in EX 1, more or less: the B♭ is considerably lower, the F♯ considerably higher than those used in most tonal music. (Since frequency is inversely proportional to length in a vibrating air column or string, the overtone series is formed by successive integral *division* of the length – forcing the string or air column to vibrate in halves, thirds, quarters, etc. It was not until the early 18th century that physicists discovered that these “upper partials” were actually present in a vibrating body – that a string vibrates normally not also throughout its whole length, but also simultaneously in halves, in thirds, etc. The relative strength of these upper partials determines the sound’s tone color.) It seems reasonable, then, to produce a collection of pitches suitable for singing or playing melodies – i.e., some kind of scale – by transposing the upper harmonics down into the appropriate octave, thus producing pitches whose ratios to the original fundamental are expressible as simple fractions: EX 2.

The difficulties with this procedure arise because there exist several alternative ways to compute the pitches. Consider a series of ascending perfect fifths, starting with our low C. If each bears the frequency ratio 3:2 to its lower neighbor, the ratio of the upper E to the low C is $(3/2)^4 = 81/16$: EX 3**.** But the same E, computed via the overtone series (as in EX 1) has the ratio to the fundamental 5:1 = 80/16. So the two different E’s differ by one part in 80 (a difference known as the “syntonic comma”). Put another way, the E that we use for the 5th degree in the key of A will sound noticeably sharp as the third degree of C. If we should go so far as to try to generate all 12 pitches by using this staircase of 5ths, by the time we arrive at the end – ostensibly back at C, seven octaves higher – we have the ratio $(3/2)^{12} = 129.7463$, significantly higher than the same pitch calculated by octaves, $2^7 = 128$. (This is the “Pythagorean comma”).

The final result is that there is no one method of choosing simple frequency-ratios for all the 12 chromatic pitches, in such a way that all possible 5ths, 8ves, 3rds etc. are well-in-tune – i.e., in simple frequency ratios. The various tunings used before the 18th century ("mean-tone", "Pythagorean", and others) were all very well in tune in some keys, and very out-of-tune in others. During the course of the 18th century, therefore, musicians gradually came to accept a compromise known as e*qual temperament* – from which comes the title of Bach's *Well-Tempered Clavier*, a work that required such a tuning, since it included compositions in all the keys, and which in fact did much to promote the adoption of equal temperament. The method divides the octave up into 12 absolutely equal semitones, each one bearing to its lower neighbor the ratio $\sqrt[12]{2}$ (= 1.05946....), so that the octave, $(\sqrt[12]{2})^{12} = 2$. In practical terms, this tuning spreads the out-of-tuneness equally throughout the octave, making it virtually imperceptible. The perfect fifth in the middle of a keyboard tuned in equal temperament is thus a few beats (cycles per second) flat from the acoustically pure ratio 3/2.

The 12 pitches arranged as compactly as possible form a *chromatic scale*; the starting point is of course arbitrary. Our notation system provides alternative ways to notate any pitch – e.g., D♭ and C♯ – and these are said to be "enharmonic equivalents". Which notation is used depends upon the context.

Just as in equal temperament the stack of half-steps closes on an octave, so does a stack of 12 perfect fifths (a perfect 5th is one containing 7 half-steps) circle back to the initial pitch, 7 octaves higher. This "scale" in 5ths, like a chromatic scale, contains all the 12 pitches, and is often arranged as the "circle of fifths":



(If it weren't for equal temperament, we would have instead a "spiral of fifths"). Notice that counter-clockwise it's a "circle of fourths". Notice also that no other series of repeated identical intervals (whole-step, minor or major thirds, tritones) will generate all the 12 pitches before returning to the starting pitch.

Basic major and minor scales

Most Western music written in the common-practice period uses at any one time not all 12 pitches, but rather a selection; this collection of 7 or 8 tones is fixed with respect not to pitch, but rather to relative interval structure. That is, whichever pitches are chosen, they bear to one another a fixed set of interval-relations. These collections of (usually) 7 pitches are customarily represented arranged compactly into a *scale*.

The major scale – that is, pitches from the major mode – has the following structure (EX 4): ½ -steps between 3°/4° and 7°/8°; all the rest are whole-steps. Transposed, it's still a "major scale", because the interval structure is the same – it can be thought of as a "7-note chord". When writing scales, or music made from scales, it is an important notational convention to use every line and space just once; this is equivalent to not skipping any letter-names: EX 5. (I will abbreviate scale degrees like so: 7°, 5°, meaning the seventh and fifth degrees of the scale, counting from the tonic at the bottom.)

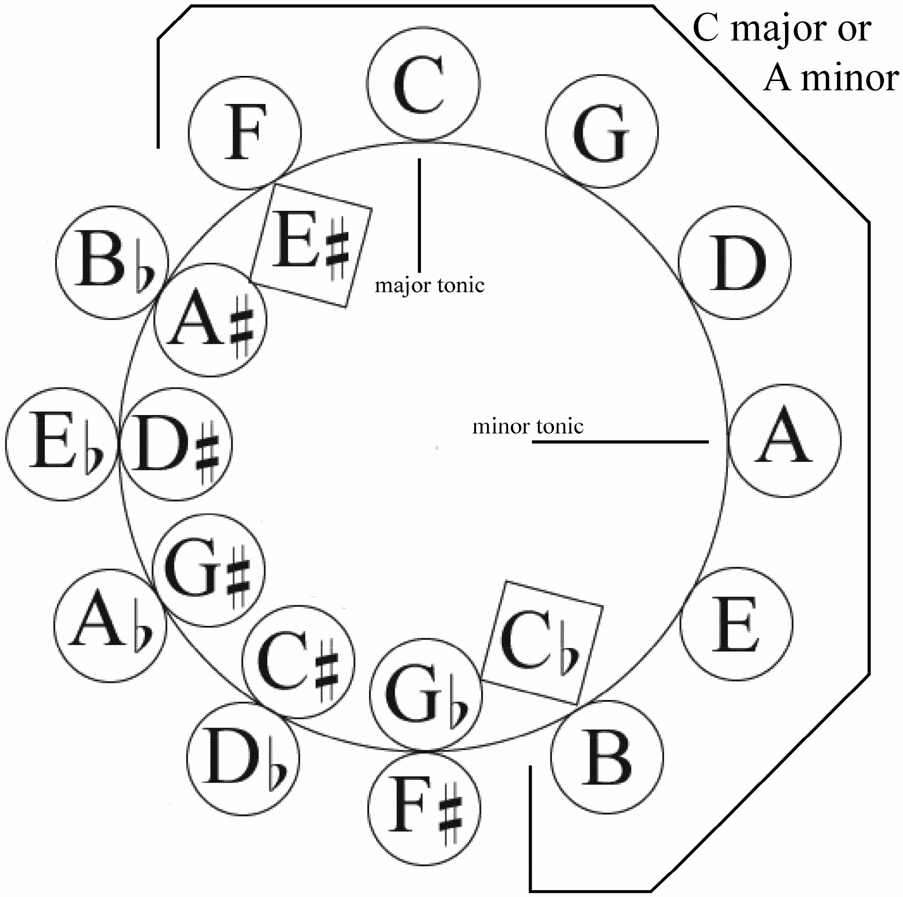
The scale degrees have *names*:

- 1° = "tonic"
- 2° = "supertonic"
- 3° = "mediant"
- 4° = "subdominant"
- 5° = "dominant"
- 6° = "submediant"
- 7° = "leading tone"
- ♭7° = "subtonic" (occurring in the minor mode)

These names have a certain symmetry: EX 6.

It is a little misleading to say that the first note of the major scale is called the "tonic"; more precisely, the notes of the major mode are customarily arranged into a scale with the tonic at the top and bottom. That is, music is made not from scales, but from collections of pitches which, to a listener who knows the style, imply one or more tonics. The keys of C major and G minor are collections of pitches used in a certain way, not just scales as such.

The circle of fifths provides a convenient way to diagram abstractly, without reference to specific pitches, the relation of the notes in the major mode. Out of the 12 pitches on the circle, it can be seen that the pitches in, say, C major form a set of 7 adjacent pitches on the circle:



Transposing this collection is simply equivalent to rotating it around the circle; thus we can say that the major mode can be "generated by" a given note, plus 6 transpositions up (or down) a perfect 5th. Notice where the tonic goes in such a representation. The furthest "harmonic distance" on the diagram is across the middle, a tritone (6 half-steps), from the 4° to 7° in major.

The pitches in the minor mode, with some qualifications, form exactly the same collection on the circle of 5ths; it's just that a different note is chosen as the tonic. When these pitches are arranged in a scale, with the minor tonic at the ends, we have the "pure" or "natural" minor scale (EX 7). The half-steps here fall 2°/3° and 5°/6°. Two keys, one major and the other minor, which share the same pitch-repertory (and thus the same key-signatures) but which have, necessarily, different tonics, are *relative* major and *relative* minor with respect to one another: the relative minor of C major is A minor. Notice that the minor tonic is a minor 3rd (3 half-steps) below the tonic of the relative major; it is the 6° of the major scale. Conversely, the major tonic is the 3° of the minor scale, a minor third above the minor tonic.

We can build a similar natural minor scale starting with C as the tonic, of course, and the key implied, C minor, is said to be the *parallel* minor to C major: EX 8. Notice that the 3°, the 6°, and the 7° are flattened from their position in major. Thus parallel major and minor scales have the same tonics, but different pitch collections (and thus different key signatures). *Relative* major and minor keys, to repeat, have different tonics, but the same basic collection of pitches, and same key signatures.

The natural minor scale, however, is rarely used as such in tonal music. At least since the Renaissance, composers have found it desirable to raise the 7° in minor when it proceeds melodically to the tonic, to provide (as in major) a half-step leading tone (LT). (In early music this was frequently done in performance, but not in the notation, a practice referred to as *musica ficta*.) A minor scale written with the normally lowered 3° and 6°, but with the raised 7°, is called the *harmonic minor scale*: EX 9. Its characteristic feature is the augmented second (3 half-steps) between the (lowered) 6° and the raised 7°. This augmented second is an awkward interval melodically, at least for average singers accustomed to Western tonal music, and so very often a melody line in minor will use a scale that, ascending, raises the 6° as well as the 7°, but changes its form descending, flatting both the 6° and the 7°. This is called the *melodic minor scale*. Notice that its descending form is identical to the "natural" minor scale: EX 10.

The minor mode as used in the common-practice period is thus variable in its upper tetrachord; alternative forms of the 6° and 7° are used depending upon the context. Notice that in every minor scale, the 3° is flattened; this is the most characteristic difference from the major mode. Most of the time, the 6° is also lowered.

Tendency tones; diatonicism

Certain notes of the scales seem to have "directional tendencies". Most pronounced is the LT, which has a marked tendency to rise to (be succeeded by) the tonic. Less compelling is the tendency of the lowered 6° to descend to the dominant, and 4° in major to descend to 3°. Notice that these all involve the scalar half-steps. But we will encounter many violations of these "tendencies".

Music written with major and minor scales is often called "diatonic" – from the characteristic scale-step, a whole-tone (2 semitones) – as opposed to "chromatic" music, using many half-steps and accidentals. Often the word "diatonic" means specifically the pitch collection of the major scale and that of the medieval modes, the seven "white notes" on the piano or their transposed equivalents.

Keys and key signatures

To transpose by a perfect fifth the pitch-repertory of a given key is equivalent to rotating it one notch on our circle-of-fifths diagram: clockwise = up a fifth, counter-clockwise = down a fifth. Naturally the major and minor tonics also move by 5ths. As we move the set of pitches, beginning with the C-major collection, we notice that "to the sharp side" – clockwise – we pick up a sharp, F♯; another notch the same direction adds C♯. This is the order in which sharps are added to the key signature of a piece, as the tonic is transposed upward by 5ths: the key of one sharp is of course G, 2 sharps is D, 3 sharps is A, etc. The sharps are written into the key signature in that order: for F♯ major (or D♯ minor) the signature reads, from the left, F♯, C♯, G♯, D♯, A♯, and E♯. As it thus happens, the *last* (rightmost) sharp in the key signature is always the 7° (LT) of the major scale, and also the 2° of the minor tonic. The *first* (leftmost) sharp will always be F♯.

Analogously, transposing the keys down a fifth is "going to the flat side", counter-clockwise in our diagram, and this is the order in which flats are written into the key signature: for the key of G♭ major or E♭ minor, the signature would read B♭, E♭, A♭, D♭, G♭, and C♭. The last (rightmost) flat in the key signature will be the 4° of the major tonic, and the 6° of the minor tonic.

Considered as a circle of keys, then, the circle of 5ths adds sharps clockwise, flats counterclockwise.

Notice the symmetry: in raising a key a fifth, the previous 4° is sharped and becomes the 7°; going down a fifth, the previous 7° is flatted and becomes the 4°. One can therefore say that the 4° and 7° are at the "tonal extremes" of the diatonic pitch-repertory.

Remember that in minor keys, the raised 7° is NOT in the key signature, but must be added in the music using accidentals; similarly with the raised 6°, when it occurs. (Somewhat similarly, in Baroque music it is common to find pieces with key signatures missing one of the usual accidentals, which is then added systematically throughout the piece.)

The keys toward the bottom of the circle of fifths are as conveniently written either as flat keys or as sharp keys:

C♭ major (A♭ minor), 7 flats ⇔ (is equivalent to) B major (G♯, minor), 5 sharps
G♭ major (E♭ minor), 6 flats ⇔ F♯ major (D♯ minor), 6 sharps
D♭ major (B♭ minor) , 5 flats ⇔ C♯ major (A♯ minor), 7 sharps.

These keys are "enharmonically equivalent"; which way they are notated depends upon convenience and context. Of course, all the keys have enharmonic equivalents; E♭ major, with 3 flats, could be (but is never) rather sadistically re-written as D♯ major, with 9 sharps, including E♯, B♯, F𝄪, and C𝄪.

To summarize keys and key-signatures: sharps are added to the key-signature in order of ascending 5ths, starting with F♯. Each new sharp raises the tonic a fifth, and represents the 7° in major (2° in minor) of the new key. Flats are added in order of descending 5ths, starting with B♭; each new flat lowers the tonic a fifth, and is the 4° of the major scale (6° of the minor) in the new key. Transposition of keys clockwise on the circle of 5ths adds sharps or subtracts flats; counterclockwise transposition adds flats or subtracts sharps.

On the naming of intervals:

Intervals are first named numerically, by counting letter names (or equivalently, lines and spaces) from one note to another, inclusive: an interval is defined by two notes, so one counts both ends.

These numbers are then qualified according to size (in half-steps, or semitones) by the terms "perfect" (in the case of unisons, 4ths, 5ths, and octaves), "major" or "minor" (in the case of seconds, thirds, sixths, and sevenths).

An interval bigger by a half-step than a perfect or a major interval is "augmented"; one smaller by a half-step than a perfect or a minor interval is "diminished". (One can also encounter "doubly augmented" or "doubly diminished" intervals, but we're not going to consider these now.) The augmented fourth and the diminished fifth are both sometimes referred to as a "tritone".

Notice: there is no such thing as a "major fourth" or a "minor fifth" or a "perfect third" or "perfect seventh": "perfect" only applies to unisons, fourths, fifths, and octaves: "major" or "minor" applies only to seconds, thirds, sixths, and sevenths.

Notice also: the terms “major” and “minor” mean simply “larger” and “smaller”; they have nothing directly to do with major or minor scales or keys.

Here is a table of interval names and semitone sizes:

name:	unison	second	third	fourth	fifth	sixth	seventh	octave
½ -steps: 0	**perfect**	*diminished*						
1	(*augmented*)	minor						
2		major	*diminished*					
3		*augmented*	minor					
4			major	*diminished*				
5			*augmented*	**perfect**				
6				*augmented*	*diminished*			
7					**perfect**	*diminished*		
8					*augmented*	minor		
9						major	*diminished*	
10						*augmented*	minor	
11							major	*diminished*
12							*augmented*	**perfect**
13								*augmented*

Some examples of each of these intervals can be seen in EX 11.

Two intervals spelled differently, and thus with different names, but with the same numbers of half-steps, such as the major third and the diminished fourth, will of course sound the same on the piano. Pitches, intervals, chords, and keys that are spelled differently but sound alike in this way are all known as “enharmonic equivalents” – a kind of musical homonym.

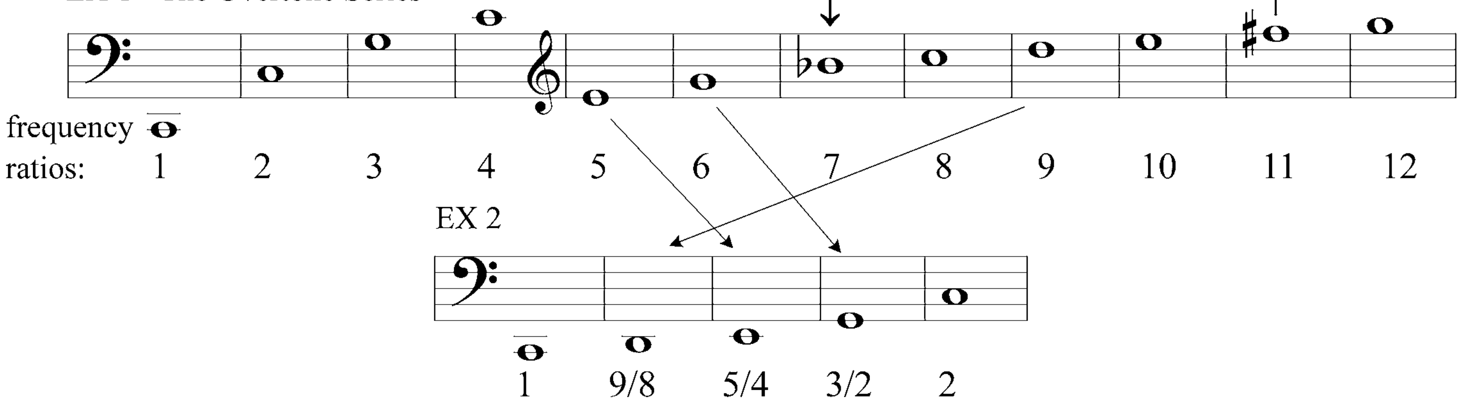
Compound intervals are intervals greater than an octave, and are harmonically similar to the interval formed by transposing the upper note down an octave (thus subtracting 7 from its name). Thus the $10^{th} \Leftrightarrow 3^{rd}$, the $4^{th} \Leftrightarrow 11^{th}$, $13^{th} \Leftrightarrow 6^{th}$, etc. The ninth, however, is usually referred to as such (rather than as a second), since in certain chords it is an important component.

Inverting an interval means transposing the lower note of an interval (in its most compact form) up an octave, so what was the lower note is now the upper. Upon this operation, unisons become octaves, seconds become sevenths, thirds become sixths, fourths become fifths, and all vice-versa. Perfect intervals invert to perfect intervals, but the other “qualities” invert: an augmented interval inverts to a diminished one, a major interval inverts to a minor one, and vice-versa. Thus the inversion of a perfect fifth is a perfect fourth, the inversion of an *augmented* fourth is a *diminished* fifth, and the inversion of a *major* second is a *minor* seventh. Intervals related by inversion are sometimes called “complementary”, and add up to an octave; a major sixth is the “complement” to a minor third.

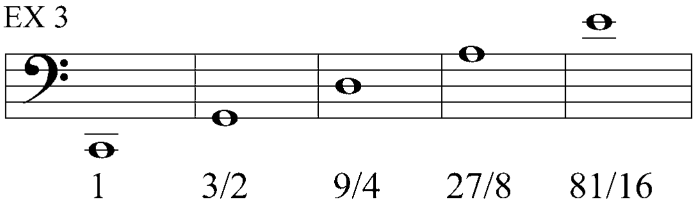
Interval consonance and dissonance: in traditional counterpoint, perfect intervals are consonant, *except for* *the fourth with the bass*; the major and minor thirds and sixths are the "imperfect consonances". All the other intervals are dissonant. We shall use this list of consonant and dissonant intervals to explain certain features of chord behavior, but we will notice also at certain times the dissonances being treated much more freely in 18th-century harmony than in 16th-century counterpoint. It should be obvious that the terms "consonance" and "dissonance" refer not to a degree of "pleasantness" or "unpleasantness", as was occasionally taught, but rather to a tendency within a traditional tonal context toward stability or instability, a tendency to move or not. While consonance and dissonance are presumably not entirely independent of the acoustics of the various intervals, equally important is simply the way composers of tonal music have chosen to treat these intervals.

Harmony Chapter 1 - Examples

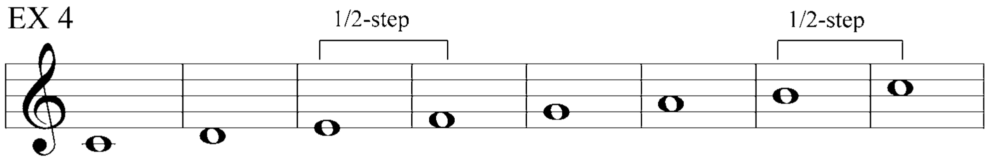
EX 1 - The Overtone Series



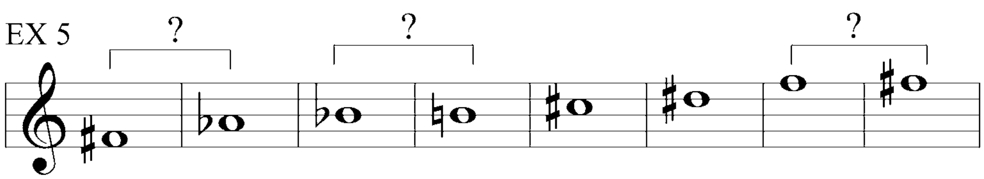
EX 3



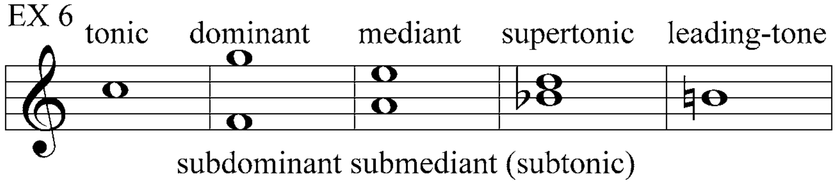
EX 4



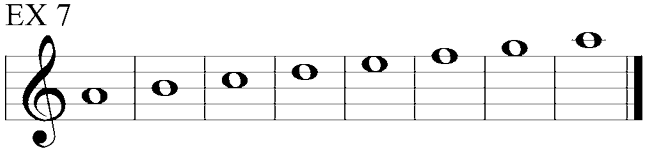
EX 5



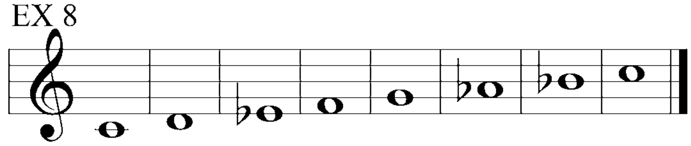
EX 6



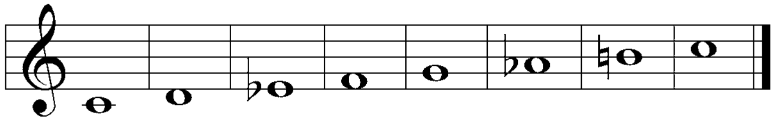
EX 7



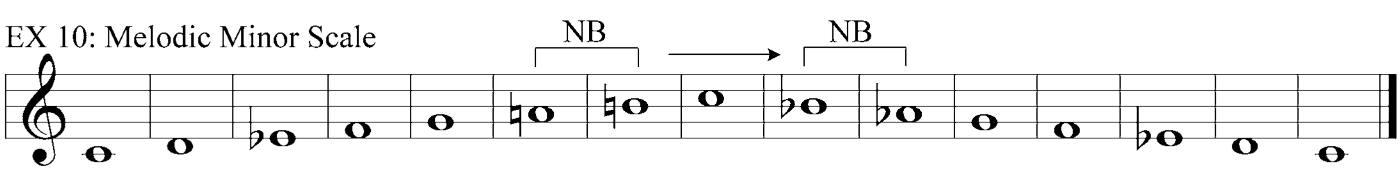
EX 8



EX 9: Harmonic Minor Scale



EX 10: Melodic Minor Scale



EX 11: Interval names, *with some examples*